PS Complexity of Polynomial-Time Problems

https://www.cosy.sbg.ac.at/~sk/courses/polycomp/

Exercise sheet 1 Due: Sunday, October 29, 2017

Total points : 40

Prove all your claims!

Reminder: In the lecture we have introduced the *Orthogonal Vectors Hypothesis*:

OVH: Given two sets $A, B \subseteq \{0, 1\}^d$ such that $|A| = |B| = n$, there is no algorithm running in time $O(n^{2-\epsilon} \cdot \text{poly}(d))$ (for any constant $\epsilon > 0$) which decides whether there exist $a \in A$ and $b \in B$ such that a and b are orthogonal.

Exercise 1 (*10 points)*

Consider the following variant **OVH'** of **OVH**:

OVH': Given a set $A \subseteq \{0, 1\}^d$ such that $|A| = n$, there is no algorithm running in time $O(n^{2-\epsilon} \cdot \text{poly}(d))$ (for any constant $\epsilon > 0$) which decides whether there exist $a, a' \in A$ such that a and a' are orthogonal.

Show that **OVH'** and **OVH** are equivalent.

Exercise 2 (*10 points)*

Design an algorithm for the orthogonal vectors problem with running time $O(2^d nd)$.

Exercise 3 (*10 points)*

Given two sets $A, B \subseteq \{0, 1\}^d$ such that $|A| = |B| = n$, construct a Boolean formula φ of size $O(nd)$ such that φ is satisfiable if and only if there exist $a \in A$ and $b \in B$ such that a and b are orthogonal.

Exercise 4 (*10 points)*

Consider the **Max Inner Product** problem: Given two sets $A, B \subseteq \mathbb{Z}^d$ such that $|A| = |B| = n$, find the value of the maximum inner product between any vector a from A and b from B, i.e, compute the quantity $\max_{a \in A, b \in B} \langle a, b \rangle$. Show that, assuming **OVH**, there is no algorithm for **Max Inner Product** with running time $O(n^{2-\epsilon} \cdot \text{poly}(d))$ for any constant $\epsilon > 0$.