PS Complexity of Polynomial-Time Problems

https://www.cosy.sbg.ac.at/~sk/courses/polycomp/

Exercise sheet 5 Due: Sunday, January 15, 2018

Total points : 40

Prove all your claims!

Exercise 1 (*10 points*)

In the $\mathbf{X} + \mathbf{Y}$ problem, we are given two sets of integers X and Y of size $|X| + |Y| = n$ and are asked to decide if the set $X + Y := \{a + b \mid a \in X, b \in Y\}$ has size $|X + Y| = n^2$, i.e., if the pairwise addition created no duplicates.

Show that if $\mathbf{X} + \mathbf{Y}$ can be solved in them $O(n^{2-\epsilon})$ for some $\epsilon > 0$, then **3SUM** can be solved in time $O(n^{2-\delta})$ for some $\delta > 0$.

Exercise 2 (*10 points*)

The problem **3SUM** can be generalized to k-**SUM** as follows: Given k sets $A_1, A_2, \ldots, A_k$ of n integers each, are there $a_1 \in A_1, a_2 \in A_2, \ldots, a_k \in A_k$ such that $a_1 + a_2 + \cdots + a_k = 0$?

Give a k-**SUM** algorithm with running time $O(n^{\lceil \frac{k}{2} \rceil} \log n)$ for constant k.

Hint: Which operations could introduce a factor of $\log n$ in the running time?

Exercise 3 (*10 points*)

Show that if k-**SUM** can be solved in time $n^{o(k)}$, then **3SAT** can be solved in time $2^{o(n)}$.

Exercise 4 (*10 points*)

Design your own exercise problem and write down a solution. The collected problems will be distributed as a preparation for the final exam.