What is this Quantum thing people keep talking about?

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Modern physics

Three scales of the universe

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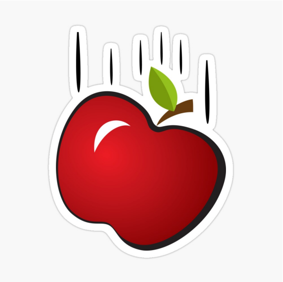

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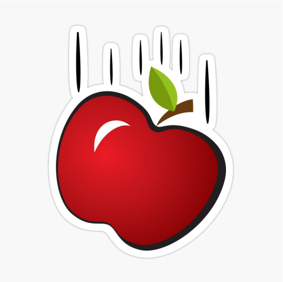

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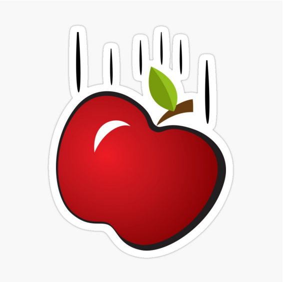

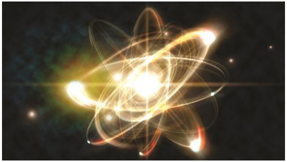

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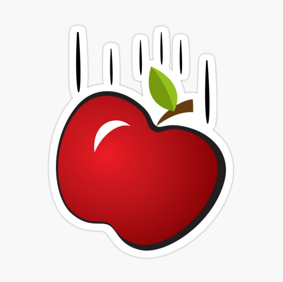
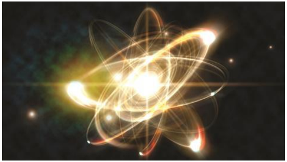
The later two are more general than classical physics.





Quantum computing?

Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.
– Richard Feynman

Computational model

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- Takeaway: Amplitudes are like probabilities but can be negative.

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- Quantum algorithms cancel the unwanted stuff and strengthen the wanted stuff.

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- If we measure we only see n classical bits, but until then all possibilities can interfere.
- Some correlations do not happen in classical probabilities:

$$\sqrt{1/2}\,|00\rangle + \sqrt{1/2}\,|11\rangle$$

This is entanglement.

Simulation

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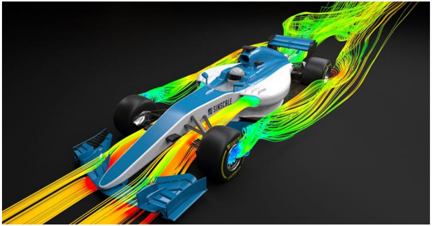

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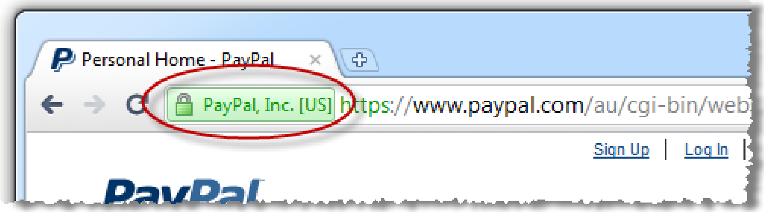
Big application: factoring large integers & breaking RSA encryption.



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- Most versatile quantum algorithm:
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  - Easy to apply to graph algorithms, NP-hard problems, optimization, ect.

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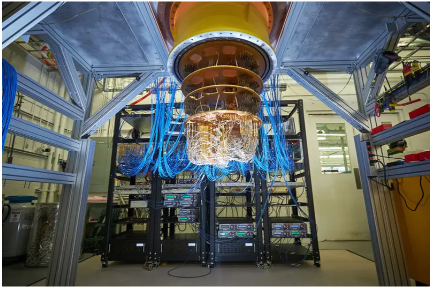

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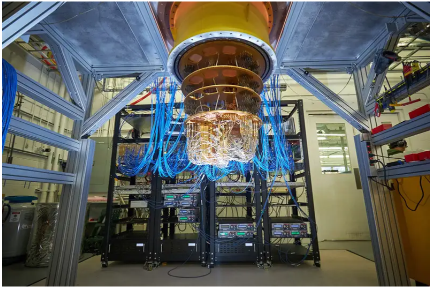

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- Roughly factor 1000 overhead. We now have ≈ 100 physical qubits on the same chip.