Fast Dynamic Distance Computation via Dynamic Spanners

Sebastian Forster

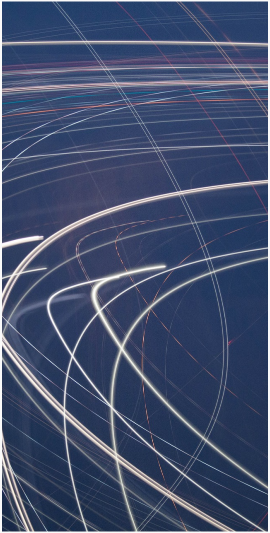
Habilitation Colloquium

University of Salzburg

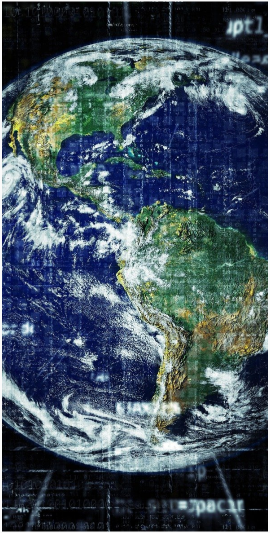
Big Data

The three V's

Volume

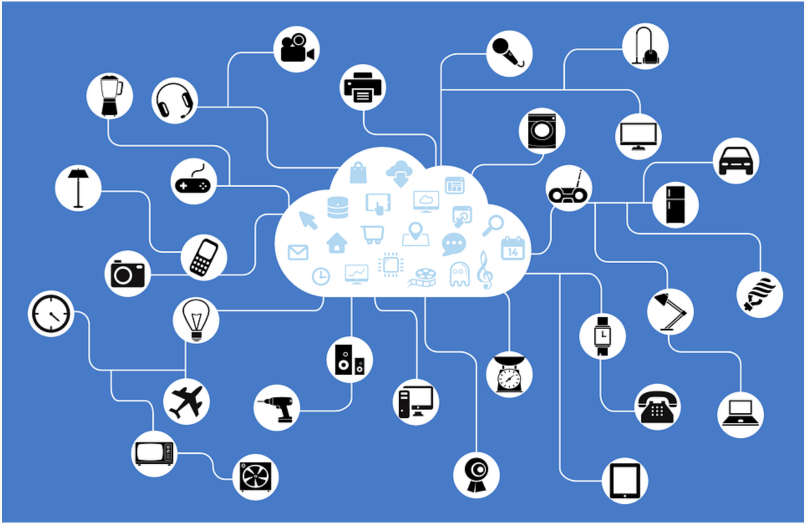


Velocity



Variety

Graphs are Everywhere



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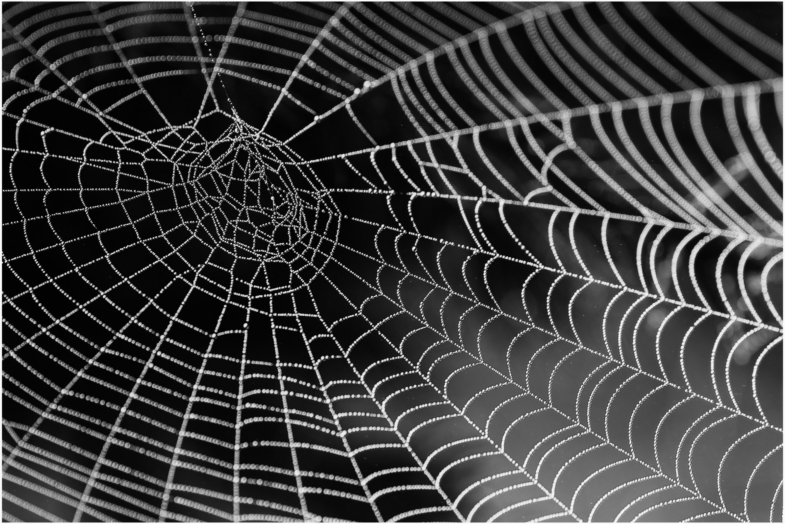
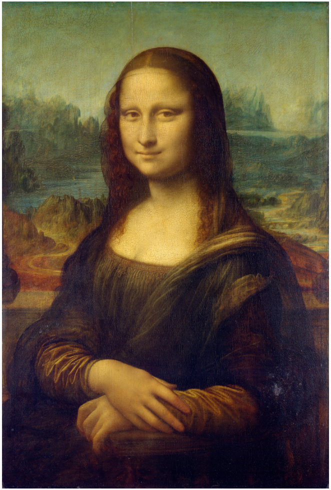


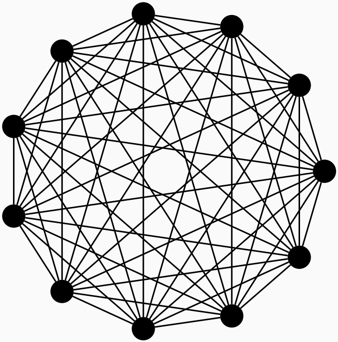
Space Reduction

"Sketching"



$\approx$

Graph Sparsification



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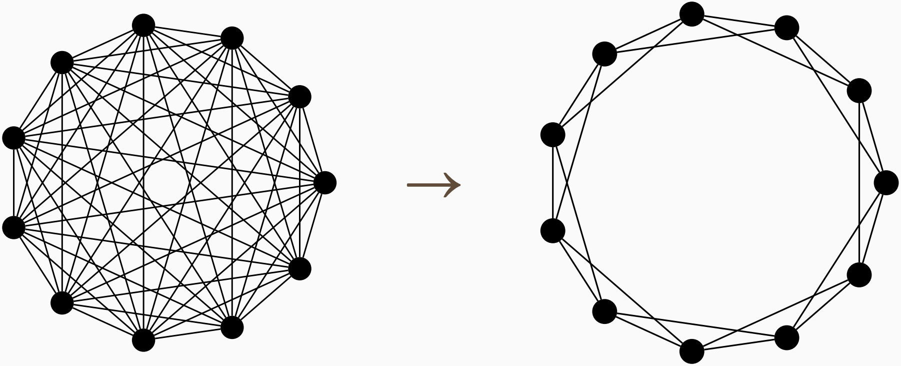

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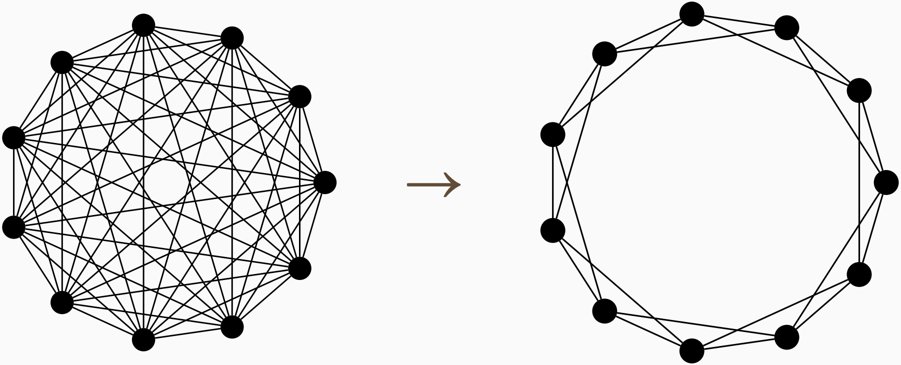

Goal: Reduce number of edges

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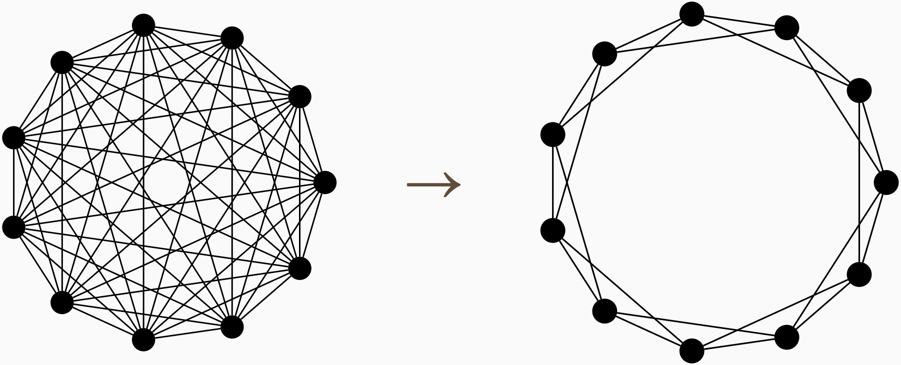

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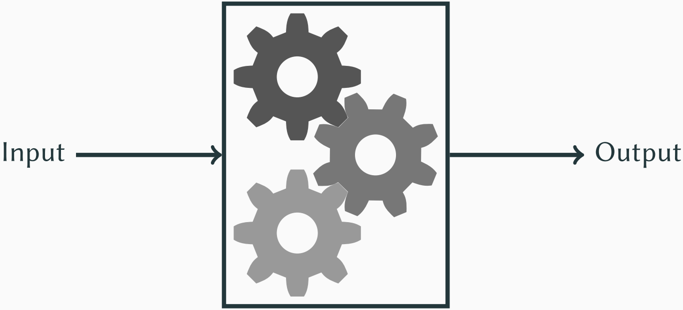
...at cost of approximation

Dynamic Algorithms

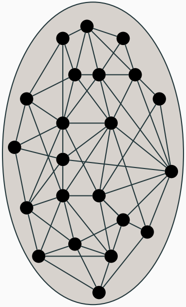
Static Approach



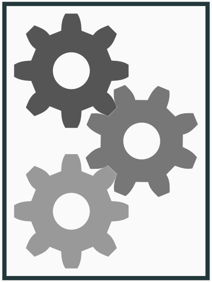
Dynamic Environments

Dynamic Sparsification

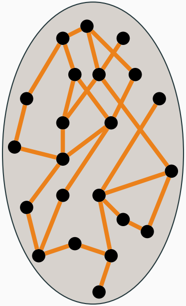
Input graph G



Algorithm



Sparsifier H



Problem Setting

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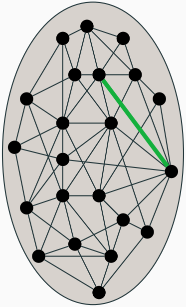

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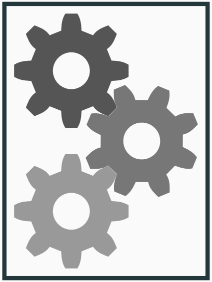

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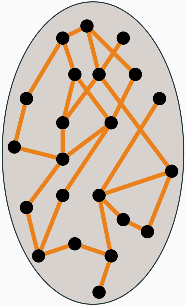

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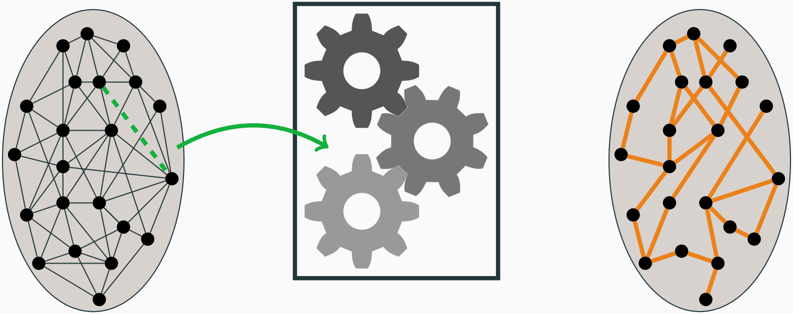

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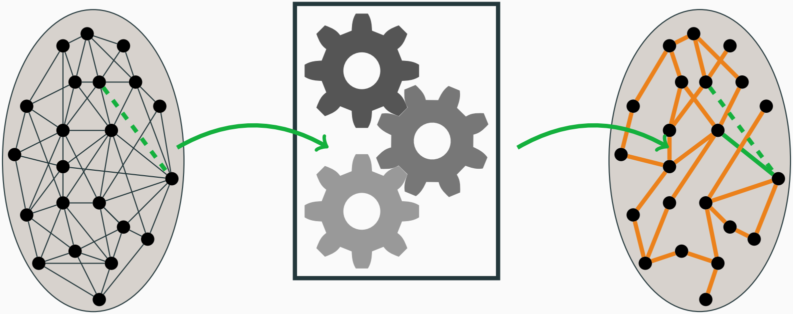

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Example 1: Distance-Preserving Sparsification

Definition ([Peleg, Schäffer '89])

A **spanner** of **stretch** t of $G = (V, E)$ is a subgraph $H = (V, E')$ such that

$$dist_G(u, v) \leq dist_H(u, v) \leq t \cdot dist_G(u, v)$$

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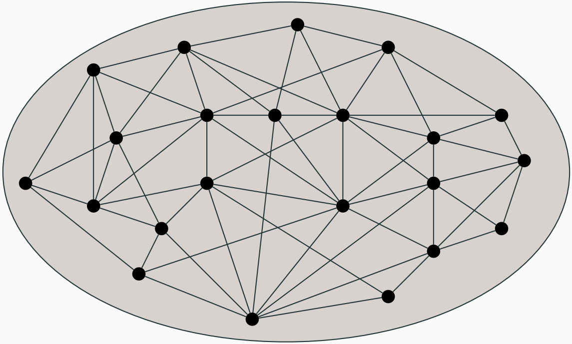

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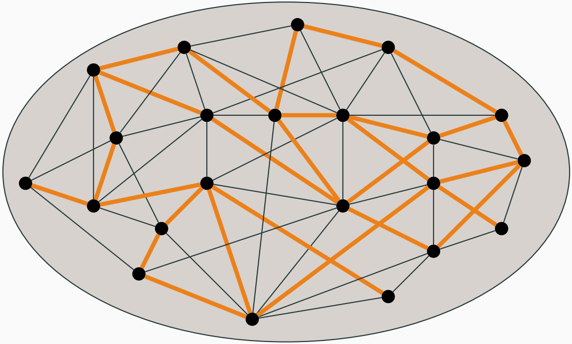

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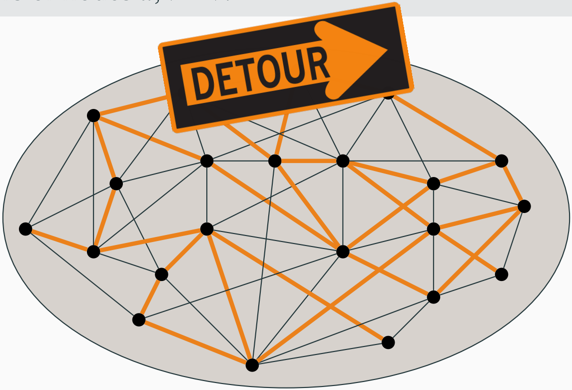

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Distributed SSSP: **boosting** approach for better approximation

[Becker, **F**, Karrenbauer, Lenzen '17]

Our Spanner Results

Theorem ([Baswana, Khurana, Sarkar '12])

For every k, there is a randomized dynamic algorithm that maintains a spanner of stretch $t = 2k - 1$

- *with* $O(n^{1+1/k} k^8 \log^2 n)$ *and* $O(7^{k/2})$ *amortized update time,*
- *with* $O(n^{1+1/k} k \log n)$ *edges and* $O(k^2 \log^2 n)$ *amortized update time.*

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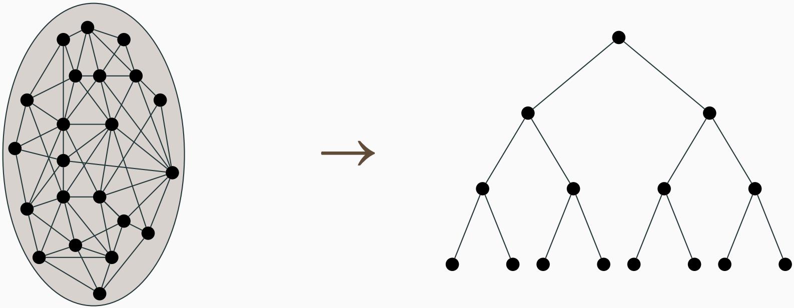
Theorem ([Bernstein, F, Henzinger '19])

For every k, there is a randomized dynamic algorithm that maintains a $(2k - 1)$-spanner with $O(n^{1+1/k} k \log^7 n \log \log n)$ edges and worst-case update time $O(20^{k/2} \log^3 n)$.

Distance-Preserving Trees

Idea: Embed distance metric into tree metric



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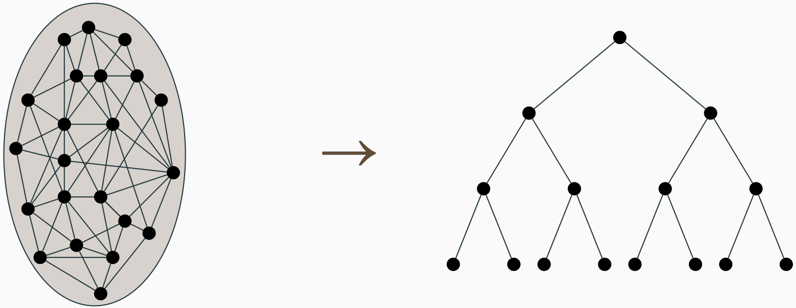

Results: First dynamic algorithms for tree embeddings:

- Average stretch [**F**, Goranci '19]
  (Recent improvement: [Chechik, Zhang '20])
- Expected stretch [**F**, Goranci, Henzinger '21]
  Applications to distance oracles and buy-at-bulk network design

Example II: Cut-Preserving Sparsification

Definition ([Benczúr/Karger '00])

A $(1 \pm \epsilon)$-cut sparsifier of G is a weighted subgraph H such that, for every cut $(C, V \setminus C)$, the edges $F := E[C, V \setminus C]$ crossing the cut have weight

$$(1-\epsilon) \cdot w_G(F) \leq w_H(F) \leq (1+\epsilon) \cdot w_G(F)$$

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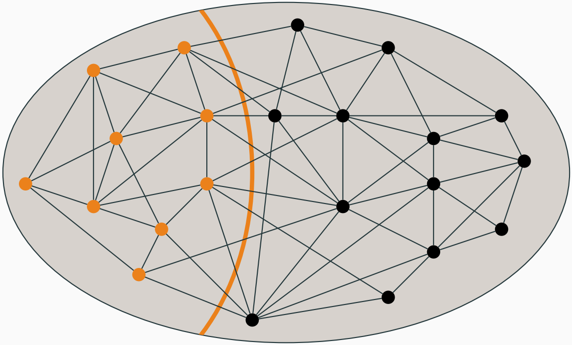

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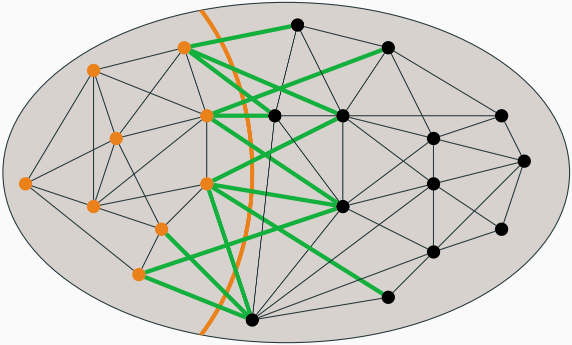

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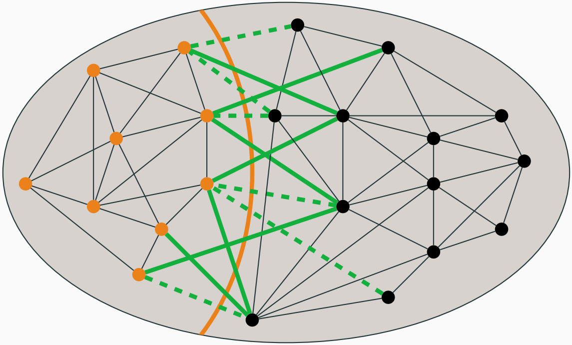

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First dynamic algorithm for this problem

Spectral sparsifier with similar guarantees at cost of amortization

Dynamic Distance Approximation

"Gold standard":

- Fully dynamic
- Worst-case update time
- Deterministic
- Meeting an update-time barrier

Towards Assumption-Free Algorithms

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Contribution

We add to this list: $(1+\epsilon)$-approximate distance approximation in unweighted, undirected graphs [van den Brand, **F**, Nazari ’22]

Our Results

Distance approximation in unweighted, undirected graphs:

Approx	Type	Update Time
$1+\epsilon$	single pair	$O(n^{1.407}\epsilon^{-2})$
$1+\epsilon$	single source	$O(n^{1.529}\epsilon^{-2})$
$1+\epsilon$	k sources	$O(n^{1.529}+kn)\cdot O(\epsilon^{-1})^{\sqrt{2\log_{1/\epsilon} n}}$
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Warm-up: *Randomized* $(1+\epsilon)$-approximate single-source

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Related work

Randomized algorithm for maintaining $(1+\epsilon, n^{o(1)})$-spanner of size $n^{1+o(1)}$ with update time $O(n^{1.529})$ [Bergamaschi et al. '21]

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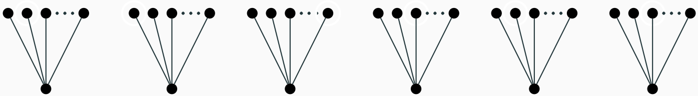

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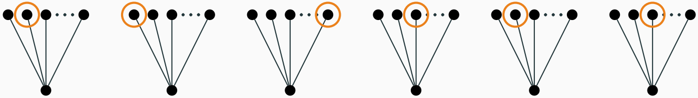

Emulator Construction

Definition

A $(1+\epsilon, \beta)$-**emulator** of $G = (V, E)$ is a graph $H = (V, E')$ such that

$$dist_G(u, v) \leq dist_H(u, v) \leq (1+\epsilon) \cdot dist_G(u, v) + \beta$$

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Emulator H has two types of edges:

- For every light node of degree $\leq \sqrt{n}$: edges to all neighbors
- For every node in hitting set: (weighted) edges to all nodes in distance $\leq \lceil 6/\epsilon \rceil$

similar to [Henzinger, **K**, Nanongkai ’13; Dor, Halperin, Zwick ’97]

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→ single-source distance on H in time $\tilde{O}(n^{1.5})$

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Subdivide any shortest path into segments of length $\lceil 6/\epsilon \rceil$ (with potentially one segment of smaller length)

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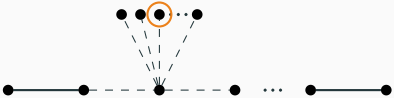

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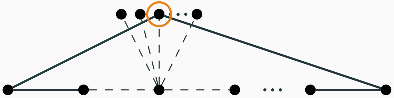

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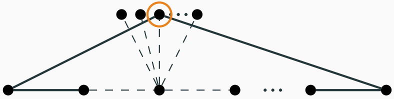

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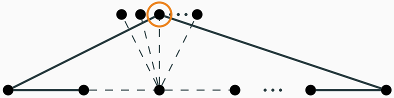

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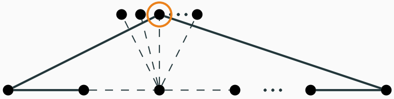

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Overall: multiplicative error of $1 + \frac{\epsilon}{2}$, additive error of 2

Algebraic Data Structure

Theorem ([Sankowski '05])

Given any $0 < \delta < 1$ and any sets $A, B \subseteq V$, there is a randomized algorithm for maintaining the $S \times V$ distances up to $\leq \Delta$ with update time $\tilde{O}(\Delta(n^{\omega(1,\delta,1)-\delta} + n^{1+\delta} + |A||B|))$.

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Approximation Guarantee:

- If $d_G(s, v) \leq \lceil 6/\epsilon \rceil$: distance from algebraic data structure
- If $d_G(s, v) > \lceil 6/\epsilon \rceil$, then approximation from H becomes $(1 + \frac{\epsilon}{2})d_G(s, v) + 2 \leq (1 + \frac{\epsilon}{2})d_G(s, v) + \frac{\epsilon}{3}d_G(s, v) \leq (1 + \epsilon)d_G(s, v)$

Observations:

- Randomization not necessary in algebraic data structure for very small distances

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- Randomization not necessary in algebraic data structure for very small distances
- Hitting set for neighborhoods can be maintained with a lazy approach giving low recourse
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- Algebraic data structure can be extended to slowly changing set of nodes

Conclusion

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- For which problems can we reach the “gold standard”
- Are there “natural” separations?

Thank you!